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LETTER TO THE EDITOR

Correlation of scaled photon-counting fluctuations

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Abstract. The use of scaling for one-bit correlation processing in optical spectroscopy of nongaussian signals is discussed. A theoretical proof of its equivalence with uniform random clipping, and hence full correlation, is provided in certain cases.

The technique of digital correlation of photon-counting fluctuations has found application in several fields of science over the last few years. Rather than measuring the true photocount (intensity) autocorrelation function, it has been found to be convenient in many instances to use 'single-clipped' correlation, which is a cross-correlation between the direct photocount signal and a one-bit 'clipped' version of it (Jakeman and Pike 1969, Pike 1972). With clipping, the advantages of one-bit correlation, namely simple circuitry and high speed of operation, are obtained at little sacrifice of experimental accuracy (Jakeman *et al* 1971, Pike 1972). However, only for a few types of incident electric field statistics, the most important being gaussian statistics, can the single-clipped correlation function be simply related to the useful true correlation functions of the field.

The wide use of clipping to date has been due to the fact that in the large majority of light-scattering experiments the signal statistics are closely gaussian. However, there do exist several classes of experiment with nongaussian signal statistics. Among these are laser light near threshold (see, for example, Jakeman *et al* 1970), light scattering from particles carried by turbulent fluids (Bourke *et al* 1970, Di Porto *et al* 1969), and light scattering from small numbers of particles undergoing motion of some kind (Adrian 1972, Schaefer and Berne 1972, Schaefer and Pusey 1972). For these experiments, unlike those with gaussian light, a measurement of the intensity correlation function $G^{(2)}(\tau)$ provides additional information to the usual heterodyne measurement of the electric field correlation function $G^{(1)}(\tau)$. (This latter measurement is independent of the signal statistics.) It is therefore essential in these cases, if this additional information is required, to measure the true intensity correlation function, rather than the clipped one, to avoid the (usually unknown) distortions introduced by clipping nongaussian signals. Several methods of making such a measurement are possible in principle, among them, full digital correlation and uniform random clipping. In the latter the clipping level is selected at random, for each sample, from a uniform distribution; as will be shown later this process, under appropriate conditions, gives the true correlation function. A third method, which has been used for a number of years (Pusey and Goldburg 1971, Schaefer and Berne 1972), has commonly been assumed also to give the true correlation function. This is the technique of 'scaling' in which a one-bit signal is obtained by scaling the original photocount signal by a factor s, chosen high enough that the probability of obtaining more than one scaled count per sampling interval is negligible compared to the probability of obtaining one. As with single-clipped correlation, this one-bit signal is cross-correlated with the original one. Like clipping, scaling has the advantages, when compared to full correlation, of being a one-bit technique. In addition, for scaling the circuitry is considerably simpler than that required for uniform random clipping.

A problem of particular interest at the moment is the utilization of photon correlation techniques for velocity measurements by light scattering. In certain such experiments we have encountered nongaussian light signals together with the requirements of high-speed electronic processing. We will not go into details of this work here, but we have recently obtained promising results using scaling, and have been motivated to attempt a rigorous justification of the method. In the following, we show that scaling approximates uniform random clipping, and that therefore it provides an estimate of the true intensity correlation function. This justifies the previous use of the method, and gives us full confidence in its application for this particular type of experiment.

We first establish the relation between single-scaled and single-clipped correlation functions of photon arrivals. Consider a correlation system consisting of two channels separated in time by a delay τ, one recording the actual signal counts detected in a sample time T, the other recording only the arrival of every sth count. We assume that if two or more scaled counts are detected in a single sampling interval they will be recorded as a one, that is, we assume the scaler to be followed by clipping-at-zero circuitry. After a period of time $\mathscr{T}$, $ms+r$ counts will have been detected with m recorded in the scaled channel and a remainder $r \leqslant s-1$. If $q(r)$ is the probability distribution of the remainder, the probability of recording one or more counts in the sample interval following $\mathscr{T}$ in the scaled channel is

$$\sum_{r=0}^{s-1} q(r) \sum_{n=s-r}^{\infty} p(n;T) \tag{1}$$

where $p(n;T)$ is the probability of counting n photons in the time T. The joint probability of recording one or more counts in the scaled channel and m counts in the other channel is thus

$$\sum_{r=0}^{s-1} q(r) \sum_{n=s-r}^{\infty} p(n,m;\tau;T) \tag{2}$$

where $p(n,m;\tau;T)$ is the joint probability of counting n photons in the sample interval T at time t and m photons in a similar interval at time $t+\tau$. The correlation function of photocounts scaled in one channel may then, with a little algebraic manipulation, be written

$$G_s^{(2)}(\tau) = \sum_{k=0}^{s-1} q(s-k-1) G_k^{(2)}(\tau) \tag{3}$$

where

$$G_k^{(2)}(\tau) = \sum_{m=0}^{\infty} \sum_{n=k+1}^{\infty} mp(n,m;\tau;T) \tag{4}$$

is the single-clipped photocount correlation function. Equation (3) has also been obtained by Koppel (1972, private communication). Scaling in one channel, therefore, averages the single-clipped correlation function over a finite distribution of clipping levels. Intuitively we might expect $q(r)$ to be uniform, in which case scaling becomes equivalent to uniform random clipping over the same distribution of clipping levels.

In this case equation (3) could be written

$$G_s^{(2)}(\tau) = \frac{1}{s} \sum_{m=0}^{\infty} \sum_{n=0}^{s} mnp(n, m; \tau; T) + [G_k^{(2)}(\tau)]_{k=s}. \tag{5}$$

If s is now chosen large enough that the second term in equation (5) is negligible compared to the first, $G_s^{(2)}(\tau)$ becomes proportional to the full photocount autocorrelation function. The uniformity of $q(r)$ is confirmed by the following analysis.

For $\mathscr{T} \gg \tau_c$, the coherence time of the light, the distribution of counts arriving within $\mathscr{T}$ will be approximately poissonian. The probability of finding a remainder in excess of an integral number s of scaled counts will therefore be

$$q(r) = \exp(-\bar{N}) \sum_{l=0}^{\infty} \frac{\bar{N}^{ls+r}}{(ls+r)!}, \tag{6}$$

where $\bar{N} = \bar{n}\mathscr{T}/T$ and $\bar{n}$ is the mean number of counts per sample time. For $s > 1$ equation (6) may be rearranged, without approximation, to give

$$q(r) = \frac{1}{s}\left\{1 + \exp(-\bar{N}) \sum_{k=1}^{s-1} \exp\left(\bar{N}\cos\frac{2\pi k}{s}\right) \cos\left(\frac{2\pi k(s-r)}{s} + \bar{N}\sin\frac{2\pi k}{s}\right)\right\}. \tag{7}$$

The second factor in this expression decreases rapidly for large $\bar{N}$ and a good approximation to the distribution, for $s > 2$, is given by

$$q(r) \simeq \frac{1}{s}\left[1 + 2\exp\left\{-\bar{N}\left(1 - \cos\frac{2\pi}{s}\right)\right\} \cos\left(\frac{2\pi r}{s} - \bar{N}\sin\frac{2\pi}{s}\right)\right]. \tag{8}$$

For $s \leqslant 4$, equation (8) is uniform to better than 1% for $\bar{N} > 5$. In the more interesting case of larger s, this degree of uniformity can only be obtained for $\bar{N} > 5s^2/2\pi^2$.

Although the most important applications of scaling are for nongaussian light, we can obtain helpful analytical results by considering scaling of gaussian light. In this case, for uniform $q(r)$, equation (3) becomes

$$G_s^{(2)}(\tau) = \frac{\bar{n}^2}{s}\left[1 - \left(\frac{\bar{n}}{\bar{n}+1}\right)^s + |g^{(1)}(\tau)|^2\left\{1 - \frac{1+\bar{n}+s}{1+\bar{n}}\left(\frac{\bar{n}}{\bar{n}+1}\right)^s\right\}\right], \tag{9}$$

where $g^{(1)}(\tau)$ is the normalized electric field correlation function. For equation (9) to differ from the true autocorrelation function by less than 1% due to the cut-off at $s-1$ in the sum of equation (3), we must choose, for $\bar{n} \geqslant 1$, $s \geqslant 10\bar{n}$. For this degree of precision, the time required for $q(r)$ to become uniform following the occurrence of a scaled count is $\mathscr{T} \simeq 25\,\bar{n}T$, which, in a typical experiment, is of the order of $\bar{n}\tau_c$.

We have performed some measurements, using both scaling and clipping, of the exponential decay rate Γ of fluctuations in (gaussian) 6328 Å light scattered at an angle of 90° by a protein (haemocyanin) solution at a temperature of 24·7 ± 0·1 °C. We used a 25-channel correlator with $\Gamma T \simeq 0{\cdot}04$. For each scaling or clipping level we performed 25 ten-second runs and obtained the means and standard deviations. The results are summarized in table 1. The results for clipping are in good agreement with the computer-simulation result of about 2% (Pike 1972). However an interesting, and possibly unexpected, finding is that (at least, for gaussian light), scaling at relatively small s is essentially as accurate as clipping, which in turn (for gaussian light) is comparable in accuracy to full correlation. We can attempt to explain this finding as

Table 1. Results of single-scaled and single clipped correlation of gaussian light produced by laser scattering from a protein solution. Γ is the decay rate of the field correlation function and $\bar{n}$ is the mean number of photocounts per sample.

$\bar{n}$	Scale or clip	Level	$\Gamma\,(s^{-1})$	$\frac{\delta\Gamma}{\Gamma}$ (%)
1·04	s	10	4137 ± 79	1·9
1·04	c	0	4158 ± 76	1·8
1·04	c	1	4161 ± 93	2·2
4·4	s	40	4170 ± 87	2·1
4·4	c	4	4217 ± 57	1·4

follows: it appears that the time $\mathscr{T}(\simeq \bar{n}\tau_c)$ taken for $q(r)$ to become uniform following the occurrence of a scaled count, can be regarded as roughly the time between independent samples of the scaled correlation function. For $\bar{n} \simeq 1$, $\mathscr{T} \simeq \tau_c$ which is approximately the same as the time ($\sim 1{\cdot}8\,\tau_c$) between independent samples in full correlation (Degiorgio and Lastovka 1971). In this case, therefore, in scaled correlation we obtain independent samples about as frequently as in full correlation. Further with $\bar{n} \simeq 1$ and $s = 10$ we are cross-correlating signals with mean rates per sample interval of about 1 and 1/10 respectively. Theoretical calculations show (Jakeman *et al* 1971, figure 3) that in such a situation the expected error in Γ should not be far from the theoretical minimum. In addition, it is possible that the 'debunching' effect of scaling (Pusey and Goldburg 1971) helps to decrease the error in Γ.

In conclusion, we have demonstrated theoretically that, under appropriate conditions, single-scaled correlation provides an excellent measure of the full photocount correlation function, regardless of signal statistics. For gaussian light this conclusion is supported by experiment. A single-clipped correlator can easily be converted for scaling by removing the reset input to the clipping gate. Alternatively the clipping circuitry can be preceded by a scaler. This latter approach allows quick selection of the optimum scaling level in any experiment: one chooses s such that the correlation counting rate is much greater for scaled-clipped-at-zero correlation, than for scaled-clipped-at-one correlation. The scaled-clipped-at-zero correlation function is then a good approximation to the full correlation function.

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